AD-A281 857

# REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

| | | |
|---|---|---|
| 1. AGENCY USE ONLY (Leave blank) | 2. REPORT DATE | 3. REPORT TYPE AND DATES COVERED<br>~~THESIS~~/DISSERTATION |

4. TITLE AND SUBTITLE
TRADE-OFF Analysis vs. Constrained Optimization with an Economic Control Chart Model

5. FUNDING NUMBERS

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
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8. PERFORMING ORGANIZATION REPORT NUMBER
AFIT/CI/CIA-
94-020

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
DEPARTMENT OF THE AIR FORCE
AFIT/CI
2950 P STREET
WRIGHT-PATTERSON AFB OH 45433-7765

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for Public Release IAW 190-1
Distribution Unlimited
MICHAEL M. BRICKER, SMSgt, USAF
Chief Administration

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)



94 7 19 023

94-22538

SCPY

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|---|---|---|---|
| 14. SUBJECT TERMS | | | 15. NUMBER OF PAGES<br>17 |
| | | | 16. PRICE CODE |
| 17. SECURITY CLASSIFICATION OF REPORT | 18. SECURITY CLASSIFICATION OF THIS PAGE | 19. SECURITY CLASSIFICATION OF ABSTRACT | 20. LIMITATION OF ABSTRACT |

94-020 D

# Trade-Off Analysis vs. Constrained Optimization with an Economic Control Chart Model

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## Why This Paper is Important

With respect to economic control chart design, this paper demonstrates that trade-off analysis following optimization has many advantages over optimization constrained by average run length. While researchers of economic design have been aware of the relatively flat cost response surface in the region of optimality, we demonstrate how this property may be exploited for improved control chart design. The flat cost surface tends to increase sharply beyond certain ARL values, causing pre-specified ARL constraints to sometimes result in substantially higher than optimal costs. Results from our trade-off approach show that, by focusing on the flat cost area, vast improvements in ARL characteristics can be obtained with minimal cost penalties.

**Abstract**

Trade-off analysis following optimization of an economic CUSUM control chart is proposed. This procedure exploits the relatively flat response surface in the neighborhood of the optimal design. Comments are made with respect to criticisms of pure economic models and the difficulties with constrained optimization are discussed. Two previously published numerical examples are used to demonstrate that, using trade-off analysis, huge gains in statistical properties are attainable with little increase in cost. By focusing on designs with improved in-control and out-of-control run length performance, false alarm rates can be reduced and the ability to quickly detect process shifts can be increased.

## Introduction

The notion of selecting control chart parameters to minimize cost over time has intuitive appeal. The cost is typically a function of many factors, among them are the cost of defects produced during an out-of-control condition prior to the control chart signal, the cost of a false alarm, and the cost of obtaining sample data. Although cost modeling of quality control systems was introduced prior to his paper, Duncan (1956) proposed the first fully economic model for single assignable causes, complete with a formal optimization methodology. His paper provided the foundation for much of the subsequent work in this area. Montgomery (1980) and Svoboda (1991) have reviewed the literature of economic control charting.

Economic control chart models are either unconstrained or constrained. Most of the early research was performed with models which are not constrained. However, Saniga (1989) used constraints with $\overline{X}$ and R charts and McWilliams (1992) has proposed a cycle duration constraint which placed an upper bound on the 95th percentile of out-of-control times. Torng, Montgomery and Cochran (1992) have investigated a constrained economic model for the EWMA procedure. The constraints usually involve specifying a minimum power and/or control interval width, or equivalently a minimum in-control average run length (ARL) or a maximum ARL for a process shift of a specific magnitude.

Woodall (1986) mentions several disadvantages associated with the economic design of control charts. Among them: 1) the parameters selected by the model may permit excessive false alarms and subsequent introduction of excess variability in the process through overadjustment, 2) economic control charts are needlessly insensitive to small shifts, 3) the use of such charts is inconsistent with the philosophy of Deming who espouses that defects should not be allowed and hence tight control of processes is required. Furthermore, models which lead to short-term profits without maintaining the controlled variable as tightly as possible about the target value violate a Deming principle and 4) economic control charts ignore the effect of management and workers efforts on the cost and time parameters. These are forceful arguments indeed, but consider the following: In today's competitive environment, cost and time must be managed while striving for continuous improvement in product quality. Economic models which limit false alarms

and provide detection of small shifts are a reality. Deming himself (Papadakis, 1985) developed a cost model for sampling inspection.

The use of statistically-constrained economic control chart models can overcome concerns 1) and 2) above. However, we feel that these models suffer from a number of drawbacks. They ignore the fact that the response surface in the region of optimality is relatively flat (see, e.g., Montgomery (1991), Torng, Montgomery and Cochran (1992)). We show that large gains in ARL properties can be achieved with only a small increase in cost by moving away from the optimal point. The ARL (or Type I and Type II error) constraints, established a priori, may lead to a solution far from the flat response surface. Furthermore, the selection of constrained values is often an arbitrary matter. For example, a particular in-control ARL value may be selected because subjectively, below that value there is a detrimental effect on employee morale. A priori ARL constraints ignore the cost penalties associated with extremely high in-control or extremely low out-of-control constraints. Even when an experienced control chart designer is available to determine "good" values, it is not clear how the cost function behaves near those design values. It is possible that slightly relaxed statistical performance constraints can result in greatly reduced costs. This paper proposes the use of a pure economic model followed by a search on the cost response surface over a region near the minimal cost. This trade-off approach considers numerous control chart configurations in the vicinity of the optimum cost, which allows the analyst and decision maker to interactively choose the best overall solution.

## Economic Statistical Approach Using Trade-Off Analysis

The design of the control chart usually involves an engineer or analyst deciding the sample size n, the sample frequency h, and the control interval limits L, for a particular type of control chart. The process is assumed to begin in-control. A search for an assignable cause begins once the sample statistic exceeds the control limits. The cycle continues until the assignable cause is removed and the process is repaired.

The model used for the application of the economic statistical trade-off method was the "unified" approach of Lorenzen and Vance (1986). Their model incorporates the three types of economic costs used in many

previous economic designs: the costs of sampling and testing, the costs associated with investigating an out-of-control signal and the search and repair of any assignable causes found, and the costs associated with the production of defective items.

The Lorenzen and Vance (LV) model was structured similarly to other economic models which solve for the expected cost per cycle divided by the expected cycle length. The objective is to determine the minimum cost design for a specified set of cost and time parameters. The optimal control chart design parameters are determined from the minimal expected cost per hour function:

$$\begin{aligned} C = & \left[ C_0 / \lambda + C_1(-\tau + nE + h(ARL2) + \delta_1 T_1 + \delta_2 T_2) \right] / ECT \\ & + \left( sY / ARL1 + W \right) / ECT \\ & + \left[ ((a + bn) / h)(1 / \lambda - \tau + nE + h(ARL2) + \delta_1 T_1 + \delta_2 T_2) \right] / ECT \end{aligned} \tag{1}$$

$$\text{where } ECT = 1 / \lambda + (1 - \delta_1) s T_0 / ARL1 - \tau + nE + h(ARL2) + T_1 + T_2 \tag{2}$$

ECT represents the expected cycle length, which is the time between successive in control periods.

Insert Table 1 here

Their approach has several attractive features that provide flexibility for a) dealing with different types of process search and/or repair conditions and b) selecting most any type of control chart. The LV model contains dummy variables that indicate whether or not a process continues during search and/or repair of the assignable cause. This single assignable cause model is developed in terms of in-control and out-of-control average run length values instead of levels of risk ($\alpha$ and $\beta$) common to many existing designs. Thus, any type of control chart that generates ARLs can be modeled using this approach. This unique feature allows for comparison of different control chart types using metrics such as cost and statistical performance.

The length of time between assignable causes is generally assumed to be an exponential random variable in economic control chart designs. The LV model is robust to deviations in the assumed in-control time distribution. McWilliams (1989) showed that substituting the Weibull distribution with varying shape and

location parameters for the assumed exponential distribution, has virtually no effect on either the optimal control chart design or the resulting cost. Banerjee and Rahim (1988) developed an approach for Weibull time between process upsets using Duncan's model.

The control chart selected for this demonstration is the two-sided analytic CUSUM. The analytic form of the CUSUM is computationally simple and focuses on deviations from the target. The CUSUM is an effective control chart procedure for detecting small shifts. It is also flexible enough to work well with size one samples, common in the chemical and process industries.

To model the CUSUM control chart using the LV model, the control interval width L used by Lorenzen and Vance was replaced by the CUSUM decision interval (we refer to as H), and reference value k. A search of possible combinations of the decision variables n, H, k, and h is conducted to find the optimal values $n^*$, $H^*$, $k^*$, and $h^*$ that minimize hourly cost. The optimization procedure uses a grid search for values of n, H, and k, and a Golden section search for h to minimize expected hourly cost. The CUSUM control chart in-control and out-of-control average run lengths were computed using the Markov chain approach described by Brook and Evans (1972). The two-sided CUSUM was used because it is important to guard against shifts in either direction (Lucas (1985).

It is recommended (see Montgomery (1991), Hawkins (1992)) to select the reference value k equal to the magnitude of the process shift ($\Delta$) divided by two. According to Chiu (1974) and Lucas (1982), this approach gives the smallest out-of-control ARL (ARL2) for a given in-control ARL (ARL1). Because our interest is in finding alternatives with improved statistical performance, we set the reference value to $\Delta/2$. We have also conducted numerous sensitivity studies on the LV model CUSUM and confirmed this recommendation (Simpson and Keats (1993)).

Although the grid search is not computationally efficient , it worked well in this case, providing the alternative designs near the optimum cost. An optimal cost was determined by using the Golden Section search on h for discrete combinations of n and H. The designs were then ordered according to cost and the ARL pairs were plotted to provide the analyst with a visual interpretation of the ARL structure of designs near optimal cost.

As was discussed earlier, the cost surface tends to be relatively flat in the region near the optimum, which provides many design alternatives within only a few percent of the optimal solution. These design alternatives tend to offer a wide range of ARL1 and ARL2 combinations. As cost increases fractionally, designs resulting in longer ARL1 and shorter ARL2 values are available. The following examples show the dynamics of the design characteristics near the values of optimal cost.

**Example 1**

We considered the example presented by Lorenzen and Vance (1986). They used a fraction defective control chart to isolate assignable causes for high readings in carbon-silicate content in castings. High levels of carbon-silicate indicated that the castings would have low tensile strength. We decided to apply the CUSUM control chart with many of the same initial cost and time parameter values. Modifications were made to the size of the shift and the fixed cost per sample. Previous analysis of CUSUM economic models by Goel and Wu (1973) showed that the size of the process shift is one of the most significant factors affecting the design parameters. Two scenarios representing different size shifts were tested. We used a small (0.25) and medium (1.25) process shift. The example variables are:

| | | |
|---|---|---|
| $\lambda = 0.03$ | $E = 0.333$ | $a = \$1.0$ |
| $T_0 = 0.333$ | $C_0 = \$115$ | $b = \$4.0$ |
| $T_1 = 0.333$ | $C_1 = \$950$ | $Y = \$975$ |
| $T_2=1.5$ | $W = \$975$ | $\Delta = 0.25, 1.25$ |

The Golden Section and grid searches discussed earlier were run on a 486 personal computer in about 30 seconds, computing not only the global minimal cost, but also the local minimal costs for given n and H values. Both the global and local minimum costs were obtained using the Golden Section search to locate the minimal cost sampling interval given fixed values of the other design parameters n, H, and k. The sample size ranged from one to 12. The decision interval values ranged from 0.5 to 6.5 in 0.5 increments, resulting in 156 different combinations of n and H.. For each shift scenario, the resulting designs were

ordered by cost and plotted. Table 2 shows the design alternatives closest to the optimal value for the 1.25 shift case.

Insert Table 2 here

Notice that deviations from optimal cost are very small and in some cases the ARL pairs offer significant performance improvement. For instance, the minimal cost ARL2 can be decreased from 7.0 to 4.7 while nearly doubling the ARL1 (393.9 to 736.0), for only a 1.3% increase in cost. The ARL2 can be reduced even further to 3.3, keeping the ARL1 at 393.9 by increasing cost 2.5%.

The plot shown in Figure 1 is a three dimensional representation of the ARL trade-offs versus cost. The data points for this illustration were obtained by applying the model to the LV example (shift of 0.75). The sample size and CUSUM decision interval were varied (n: 1-12, H: 4.0-6.5) to generate the points below. The resulting surface highlights the relative insensitivity of changes in ARL1 to changes in cost. Considerable gain in the in-control run length can be made for small cost penalties. Conversely, changes in ARL2 cause steep changes in cost. The costs increase exponentially with decreasing ARL2 for a given ARL1. This asymptotic behavior is also illustrated later in Figure 6.

Insert Figure 1 here

Figures 2 and 3 present the same type of information in a clustered bar format, which enables the designer to view the ARL structures of the chart alternatives. Figure 2 shows several designs with improved ARL1 and ARL2. For only a 1.2% cost increase, the in-control ARL can be increased 50% (41.8 to 61.5) and ARL2 can also be reduced slightly. Figure 3 is plotted using a logarithmic scale because the design alternatives can be quite large. For example, the ARL1 can be increased from the optimal cost ARL1 of 393.9 to nearly 8700 while obtaining a slightly smaller ARL2. This design costs about 6.1% more than the optimal design, but may be worthwhile in some situations.

Insert Figure 2 here

Insert Figure 3 here

**Example 2**

We used a modification of the example shown in Montgomery (1991, p. 420) to test the behavior of the trade-off approach using somewhat different input values. In this example, control of soft drink bottle thickness is monitored because the manufacturer is interested in detecting whether the wall of the glass is too thin. If this condition occurs the internal pressure generated during filling will cause the bottle to burst. The Montgomery example applied Duncan's model which assumes the process continues during search and repair of the assignable cause. We set those flags ($\delta_1$ and $\delta_2$) to one in the LV model to simulate that situation. We also assumed a nominal value for the defect cost during in-control condition. Two scenarios representing the small and medium process shifts were developed and tested. The process cost and time values are:

| | | |
|---|---|---|
| $\lambda = 0.05$ | $E = 0.0833$ | $a = \$1.00$ |
| $T_0 = 1.0$ | $C_0 = \$5$ | $b = \$0.10$ |
| $T_1 = 1.0$ | $C_1 = \$100$ | $Y = \$50$ |
| $T_2 = 1.0$ | $W = \$25$ | $\Delta = 0.25, 1.25$ |

The results of the grid search show a flatter cost surface than the LV example. Using the same grid search method of determining optimal designs for various values of n and H, the initial cost parameters from this example resulted in more design alternatives with the same percent increase in cost. For instance, within 2% of the optimal cost for the 0.25 shift LV example, 37 design alternatives were available. In this example, 63 design alternatives are options within the same 2% of optimal cost. Thus, it is possible that more significant improvements in the statistical properties of the control chart design can be made for the same fractional increase in cost. In the case of the 0.25 shift (Figure 4), the ARL1 can be doubled (16.7 to 33.9) with only a small increase in ARL2 (6.0-7.8) while cost increases just 0.9%.

Insert Figure 4 here

For the medium process shift (Figure 5), the optimal cost design ARL1 (58) can be almost doubled (111), with very small change in ARL2 (1.77 to 1.99) for a cost increase of only 0.2%. The ARL1 can be raised to 209, with an ARL2 of 1.77 for a 2.8% cost increase.

Insert Figure 5 here

The ARL improvements we have discussed are representative of the types of improvements that can be made within a few percent of the optimum economic cost. The figures displayed many options within about five percent of minimum cost. Greater improvements can be made in both types of ARL but at some point the response surface becomes steep and cost increase substantially. Figure 6 shows the relationship between costs and the two ARL types for a given example. The Lorenzen and Vance baseline case was used with a shift of 0.75 process standard deviations. To generate the ARL alternatives, we modified the decision interval (H) and sample size (n) with fixed reference value ($k=\Delta/2$), and minimum cost sampling interval (h). The general shape of the curves indicate the need to carefully select ARLs, especially in the case of ARL2. ARL2 can be improved in the immediate vicinity of the optimum from 17 to about 7 for slight increases in cost. Further improvements in ARL2 however, becomes increasingly expensive. For the in-control case, improvements in ARL1 can generally be made at either a slight increase in cost or a slight increase in ARL2, depending on the region of operation. This figure clearly shows the dangers associated with pre-specified ARLs. Pre-specifying ARL2 < 8 costs very little relative to the optimum. Requiring ARL2 < 4 costs about 30% more than optimum. This type of illustration can help the decision maker decide which cost/statistical performance parameters are best for a particular scenario.

Insert Figure 6 here

## Futher Research and Summary

The trade-off approach could be enhanced by developing a multi-objective program that allows the designer to select weights for the three objectives: minimize cost, maximize ARL1 and minimize ARL2. Ideally, the program would contain an interface so that what-if scenarios could be developed and solved so that the designer could interact to develop the "best" solution. The GRG nonlinear optimization technique is a good candidate approach. A more detailed description of this technique can be found in Luenberger (1989) or Reklaitis et al. (1983).

Similar to the economic statistical designs, the trade-off approach was developed to address the concerns mentioned in the introduction about economic designs. By focusing on designs with improved in-control and out-of-control run length performance, false alarm rates can be reduced and the ability to quickly detect shifts can be increased. Improved statistical performance with focus on achieving the largest improvement in the region of minimum cost is also consistent with Deming's philosophy of tight process controls. The charts presented in the examples enable designers to quickly understand the financial impacts of ARL improvements.

The trade-off approach provides the analyst or engineer with a view of a portion of the cost surface that may not be available under the constrained statistical economic approach. The constrained approach is certainly valid if the constraints are required for a specific reason. It is possible though, that in many instances the control chart designer may not be able to define firm statistical constraints needed to successfully monitor and detect process assignable causes.

The primary advantage of the trade-off approach over other economical statistical approaches is that it focuses the attention on the design alternatives in the immediate vicinity of the optimum. It takes some of the guess work out of constraint formulation. Instead of arriving at a single best constrained design, the analyst can prepare several options with slightly different benefits, so that the decision maker can select the best choice for the organization.

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| *Symbol* | *Input variable* |
|---|---|
| n | Sample size |
| $\lambda$ | Number of assignable causes per hour |
| $T_0$ | Expected search time when false alarm |
| $T_1$ | Expected time to discover the assignable cause |
| $T_2$ | Expected time to repair the process |
| E | Time to sample and chart one item |
| $C_0$ | Quality cost/hr while producing in control |
| $C_1$ | Quality cost/hr while producing out-of-control |
| W | Cost for search/repair |
| a | Fixed cost per sample |
| b | Variable cost per unit |
| Y | Cost per false alarm |
| $\Delta$ | Mean shift - number of standard deviations slip when out-of-control |
| $\delta_1$ | Flag for whether production continues during searches (1-yes, 0-no) |
| $\delta_2$ | Flag for whether production continues during repairs (1-yes, 0-no) |
| ARL1 | In-control average run length |
| ARL2 | Out-of-control average run length |
| L | X bar chart - number of standard deviations from control limits to center line |
| h | Hours between samples |
| k | CUSUM reference value |
| H | CUSUM decision interval |
| $\alpha$ | Probability of a type I error or probability of a false alarm |
| $\beta$ | Probability of a type II error or 1.0 minus the power of the |

**Table 1.**
**Explanation of terms**

| Cost | Cost Increase | Sample Size | Reference Value | Decision Interval | Sample Interval | ARL1 | ARL2 |
|---|---|---|---|---|---|---|---|
| ***$142.31*** | ***0.0%*** | ***1*** | ***0.625*** | ***4.0*** | ***0.35*** | ***393.9*** | ***7.0*** |
| $142.56 | 0.2% | 2 | 0.625 | 3.5 | 0.67 | 209.9 | 3.8 |
| $142.72 | 0.3% | 1 | 0.625 | 4.5 | 0.31 | 736.0 | 7.8 |
| $142.98 | 0.5% | 1 | 0.625 | 3.5 | 0.41 | 209.9 | 6.2 |
| $143.07 | 0.5% | 2 | 0.625 | 4.0 | 0.60 | 393.9 | 4.2 |
| $143.09 | 0.5% | 2 | 0.625 | 3.0 | 0.78 | 111.0 | 3.4 |
| $143.80 | 1.1% | 1 | 0.625 | 5.0 | 0.29 | 1369.8 | 6.6 |
| **$144.23** | **1.3%** | **2** | **0.625** | **4.5** | **0.55** | **736.0** | **4.7** |
| $144.50 | 1.5% | 3 | 0.625 | 3.0 | 1.04 | 111.0 | 2.6 |
| $144.85 | 1.8% | 3 | 0.625 | 3.5 | 0.89 | 209.9 | 2.9 |
| $145.10 | 2.0% | 2 | 0.625 | 2.5 | 0.97 | 58 | 2.9 |
| $145.23 | 2.1% | 3 | 0.625 | 2.5 | 1.27 | 58 | 2.3 |
| $145.24 | 2.1% | 1 | 0.625 | 3.0 | 0.51 | 111.0 | 5.4 |
| $145.26 | 2.1% | 1 | 0.625 | 5.5 | 0.27 | 2542.2 | 9.4 |
| $145.74 | 2.4% | 2 | 0.625 | 5.0 | 0.52 | 1369.8 | 5.1 |
| **$145.93** | **2.5%** | **3** | **0.625** | **4.0** | **0.83** | **393.9** | **3.3** |

**Table 2.**
**LV example with shift = 1.25**

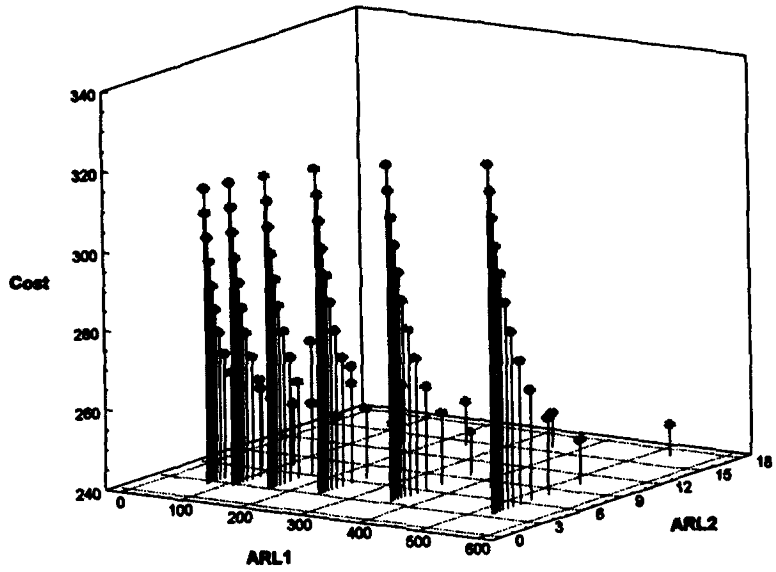



**Figure 1.**
**LV Example for Shift = 0.75**

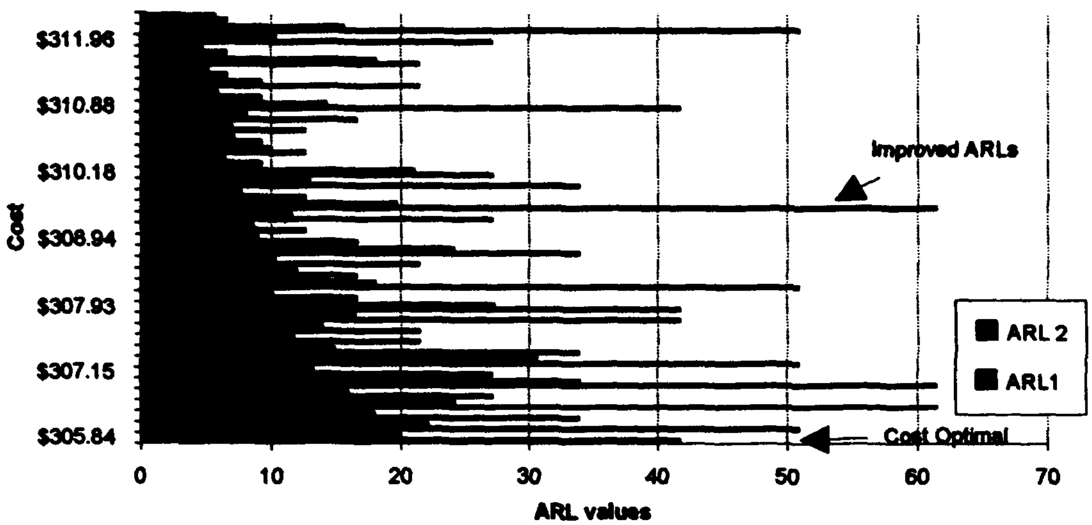



**Figure 2.**
**Example 1 for Shift = 0.25**

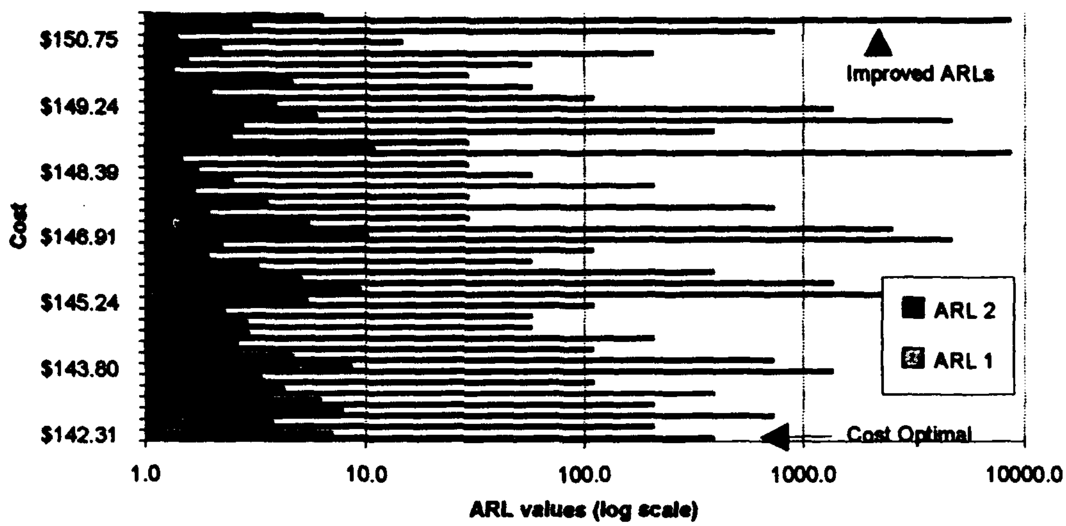



**Figure 3.**
**Example 1 for Shift = 1.25**

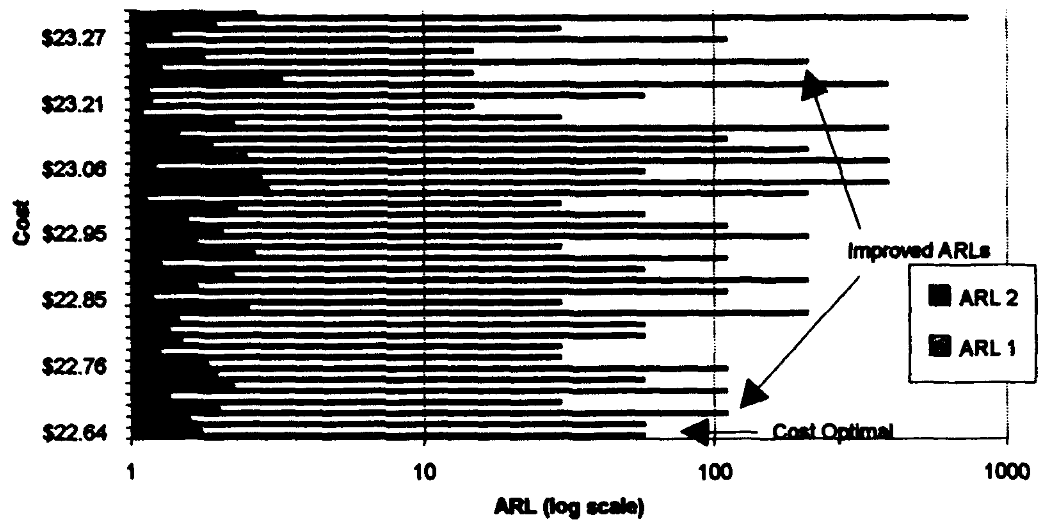



**Figure 4.**
**Example 2 for Shift = 0.25**

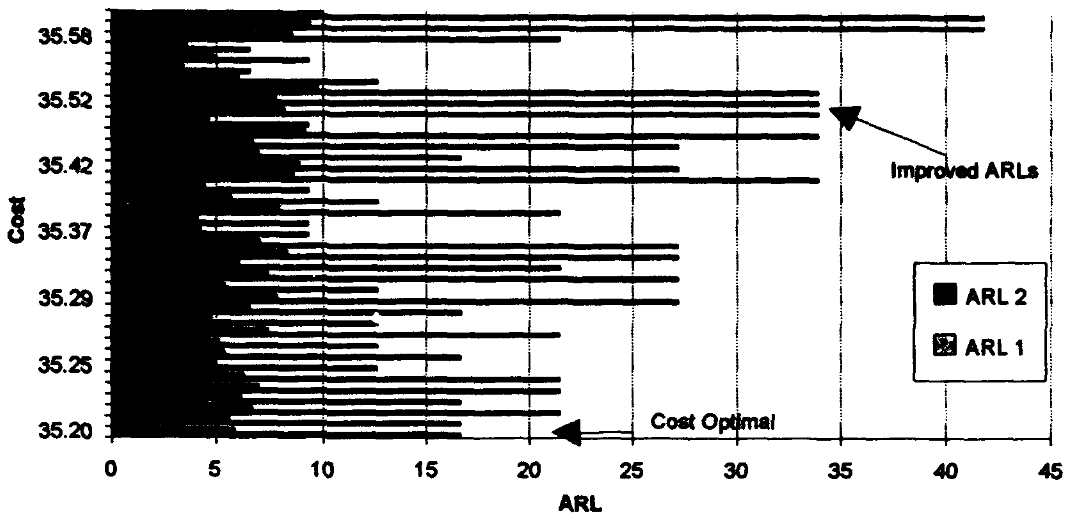
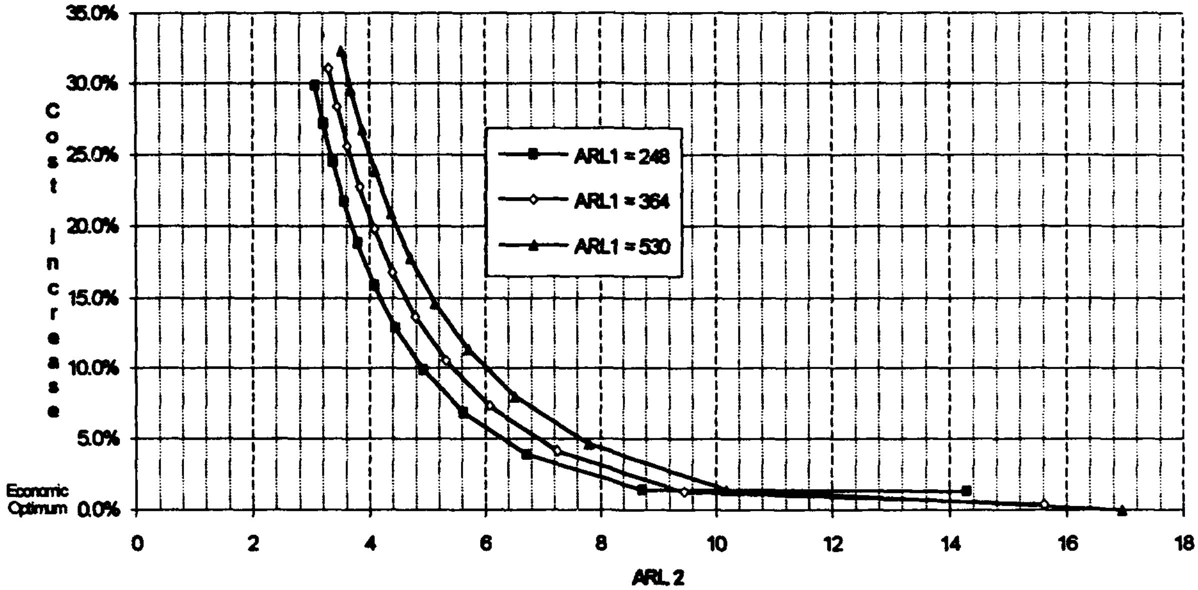



**Figure 5.**
**Example 2 for Shift = 1.25**




Assumes: Fixed reference value (delta/2), minimal cost sampling interval, variable sample size and decision interval

**Figure 6.**
**ARL Comparison for shift of 0.75**

# Sensitivity Study of the CUSUM Control Chart with an Economic Model

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## Abstract

Economic control chart designs have not been universally implemented in industry for reasons such as the parameters are too numerous and often difficult to accurately estimate. A possible solution to these problems involves performing a sensitivity analysis of the inputs to determine which parameters are significant and how parameter misspecification impacts the results Using two-level fractional factorial designs, we identify highly significant parameters in the Lorenzen and Vance economic control chart model under a Cumulative Sum (CUSUM) condition. The response variables examined are expected cost per time unit, sample size, sampling interval, control chart decision interval and reference value. A verification test and misspecification test supported our conclusions with respect the expected cost per time unit response variable. The results of the study should make industrial implementation an easier task.

## 1. Introduction

Economic considerations are often overlooked as important factors in the design and use of control charts. To monitor and maintain statistical control of a process, control charts are often designed with respect to statistical criteria only. Many times statistically optimal control charts can be more costly than a control chart whose type and design parameters are determined by the economic consequences.

Models that determine control chart parameters based on economic factors are attractive if an organization is interested in minimizing costs related to the control process. These economic models include measures of statistical performance in the total cost equation, so that the optimum cost design incorporates considerations for the level of type I and type II error. Extensive research has been conducted in the design and development of economic models. Unfortunately, little of this successful research has been adopted by the engineers in industry. One of the concerns most often expressed in attempting to apply these models in real world situations is that there are too many inputs to estimate. One method for reducing the number of terms is to choose an economic model and the appropriate type of control chart and perform a sensitivity analysis on the input variables to determine which are critical.

To help promote the practical use of economic models in industry and help bridge the gap between researchers and practitioners, we have selected a robust economic model and a robust control chart to identify the input parameters significant to a general class of problems. We apply the Lorenzen and Vance (LV) economic model to the CUSUM control chart and perform a sensitivity analysis on the model inputs. Two previously published examples are used to test for robustness of the results. A third example is used to verify the findings of the sensitivity. Finally, because the size of the process shift is an important factor, an analysis of a wide range of possible shifts is also conducted. From these analyses we determine the key factors driving cost in the LV CUSUM

model, the key factors driving the control chart decision variables, and the extent to which certain input variables may be misspecified without appreciably affecting cost.

## 2. Literature Review

As control charts became more common in industry, cost considerations became an important factor. Cost modeling of quality-control systems was introduced prior to Duncan [1], but he proposed the first fully economic model for single assignable causes, complete with a formal optimization methodology. He developed an economic model for the Shewhart control chart. His paper provided the foundation for much of the subsequent work in this area.

Although the Shewhart chart is very popular and easy to interpret, it is not able to quickly detect small process shifts. The CUSUM chart is being increasingly applied in industry (60,000 charts monitored daily by DuPont alone) because a) it can quickly detect small process shifts b) it is very effective with size one samples, important for the chemical and process industries, c) the simpler analytic form of the chart is now widely accepted, and d) the CUSUM can be combined effectively with the Shewhart chart to detect both small and large shifts [2].

The CUSUM control chart was developed by E. S. Page [3]. The scheme became popular after Barnard's article [4]. The basic form of the CUSUM for individual continuous variables is

$$S_i = \text{Max}\,(0, cY_i - k + S_{i-1})$$

where,

$S_i$ = the CUSUM value
$Y_i$ = $(x_i - T)/s$ is a transformation of $x_i$, the $i^{th}$ observation
$T$ = the target value
$s$ = an estimate of the process standard deviation
$k$ = the reference value
$c$ = a multiplier set to +1(-1) to detect increases (decreases) in the process mean.

The reference value prevents an early signaling of an out-of-control situation. The initial value of $S_i$ is usually set at zero. The current CUSUM value $S_i$ is compared with the CUSUM control limit, H. The process is deemed out-of-control when $S_i > H$. The one-sided CUSUM has a minimum value of zero and a single control limit H. As in the case of Shewhart charts, if shifts in the process mean in both directions (two-sided case) are of interest, then two separate CUSUM charts are maintained. To apply this procedure for sample averages $x_i$ is replaced with $\bar{x}_i$ and s with $s/\sqrt{n}$.

Research on economic models for CUSUM charts began when Taylor [5] first introduced a CUSUM chart economic model, but his approach required that the sampling interval and sample size be prespecified in order to solve the model. Goel and Wu [6] developed a single assignable cause model, similar to Duncan's, for the CUSUM chart. Their work also provided sensitivity analysis on some of the model parameters. Chiu [7] modified previous approaches to CUSUM economic modeling by working with the analytic form of the CUSUM instead of the V-mask version. The analytic form offers the advantages that it is easier to compute and easier for the operator to understand.

In recent years, vast amounts of successful research have been accomplished in developing economic control chart designs, but very little has actually been implemented in industry. Some of the reasons given by Saniga and Shirland [8] and Chiu and Wetherill [9] are that the mathematical models are complex, and the model input parameters are too numerous and often hard to estimate. Others have noted that the assumptions used in developing the economic models do not apply in real world situations.

The solutions to the economic model implementation problems are steadily surfacing. In part due to the renewed interest in total quality control, statistical quality control computer software is abundant and widely used, making the use of complex models relatively simple. Although the input parameters may in some cases be difficult to estimate, Montgomery [10] noted that the cost

response is relatively flat and generally insensitive to errors in parameter estimation. Reducing the number and required precision of input parameters has been studied by Montgomery [11], von Collani [12], Montgomery and Storer [13], and Pignateillo and Tsai [14].

Advances in the applicability of these models to real world situations have also surfaced. Recently proposed economic models, such as that of Lorenzen and Vance [15], are quite robust to the type of control chart used, and the assumed assignable cause distribution. Before the Lorenzen and Vance (LV) model was introduced, economic models could only be used for X-bar and fraction defective charts. Because the LV model incorporates null and specified shift average run lengths, most any control chart can be used. Deviations from the traditionally assumed exponential time between occurrences in economic designs has been studied by Hu [16], Banerjee and Rahim [17 and 18]. Specifically with regard to the LV model, McWilliams [19] found that their design is quite insensitive to the assumed distribution. He performed a sensitivity analysis on model performance for non-exponential assignable cause distributions by applying the LV model under Weibull distributions with varying shape parameters. He found that the LV model was insensitive to the Weibull family assignable cause distribution. This finding provided additional rationale for using the LV model in situations requiring a robust framework.

## 3. The Lorenzen and Vance Economic Design

The Lorenzen and Vance model provides the practitioner the most flexibility of any of the widely known single assignable cause models available. By using average run lengths instead of type I and type II errors, LV allows the analyst to choose from any type of variable or attribute control chart. The authors also included indicator variables in the model to identify whether production ceases or continues during search and/or repair, so that any possible operational scenario can be appropriately modeled.

The LV model incorporates three types of cost ratios into its formulation: 1) the cost of producing non conforming items, 2) the cost of false alarms and of search and repair of the true assignable cause, and 3) the cost of sampling. The control chart design parameters for the LV model sample size (n), sampling interval (h), and control interval width (L) are chosen to minimize the expected cost per hour function:

$$
\begin{aligned}
C = {} & \frac{C_0/\lambda + C_1(-\tau + nE + h(ARL2) + \delta_1 T_1 + \delta_2 T_2)}{ECT} \\
& + \frac{sY/ARL1 + W}{ECT} \\
& + \frac{[(a+bn)/h][1/\lambda - \tau + nE + h(ARL2) + \delta_1 T_1 + \delta_2 T_2]}{ECT}
\end{aligned} \tag{1}
$$

$$\text{where } ECT = 1/\lambda + (1-\delta_1)sT_0/ARL1 - \tau + nE + h(ARL2) + T_1 + T_2. \tag{2}$$

ECT represents the expected cycle time, which is the time between successive in control periods. Table 1 provides a description of each of the model parameters as well as definitions of other terms used in the paper.

**Table 1.**
**Input Variables**

To model the analytic CUSUM control chart in the LV framework, their control interval width term L was replaced by the CUSUM decision interval H, and reference value k. A search of possible combinations of the decision variables n, H, k, and h is conducted to find the optimal values n*, H*, k*, and h* that minimize hourly cost. The optimization procedure included a grid search on n, H, and k, and a Golden section search on h to minimize expected hourly cost. The CUSUM control chart average run lengths were computed using Brook and Evans [20] Markov chain approach. Because the matrix inversion routines were computationally time consuming,

ARL tables for various combinations of n, k, H, and shift in the process mean ($\Delta$) were developed and a file lookup technique was used for the optimization runs.

**4. Sensitivity Analysis**

By examining the cost equation for the LV model it is evident that, although the terms fully describe the economics of the control chart process, there are many parameters to estimate. The purpose of the sensitivity analysis is to determine the key drivers of cost and four control chart decision variables. We performed an experimental design and analysis of the twelve time and cost input parameters. Table 2 provides a description of each input and indicates which portion(s) of the model that variable affects. The response variables were expected cost per time unit, CUSUM decision interval, CUSUM reference value, sampling interval, and sample size. The designs were developed and run against two different previously published examples. We used the example from the LV paper [15] and an example from Montgomery [21, p. 420] to provide a diverse set of realistic scenarios. We also developed a third scenario, somewhat different from the first two to verify the results of the two example sensitivity analyses. We modified the variables in groups (significant versus nonsignificant) and compared the cost response variability for each group.

The designs for the first three scenarios included low, center and high levels for process shifts of 0.25, 0.75 and 1.25. An additional design was developed to determine whether the important variables changed if larger process shifts were assumed. This design used process shift levels of 1.25, 1.75 and 2.25. We used the LV example input levels for this experiment and compared the results to the smaller shift scenario from Example 1.

**Table 2.**
**Input Variables**

A resolution IV $2^{12-6}$ design was selected so the main effects could be estimated by not being confounded with the two factor interactions. A range of +/-30% for the baseline variable values was used to calculate the high and low levels. Because the equation to calculate expected cost was deterministic, a single design replicate and single center point were run. The deterministic nature of the response required us to use a heuristic approach for identifying significant variables. Significant variables were determined by inspection of the normal probability effects plots. Higher order interaction terms were pooled to provide an estimate of error. Significant main effects were identified in an effort to identify the most parsimonious model. The proposed effects were used to develop an analysis of variance (ANOVA) model and the effect estimates and standard errors were calculated. Typically a cutoff point of p=0.05 is used to determine significance. In this situation however, because the cost model is deterministic, there is no noise term in the ANOVA other than the higher order terms. As a result, the standard errors of the effect estimates tend to be very small and most of the main effects and two factor interactions were significant at the five percent level. In the interest of parsimony and dimension reduction, only the major contributors were selected for inclusion into each model.

Example 1

The first scenario used in the sensitivity analysis was an example used by Lorenzen and Vance [15] when they introduced their economic model. They considered the economic implications of the use of a fraction defectives chart (p-chart) in a foundry operation. The purpose of the control chart was to isolate assignable causes for high readings in carbon-silicate content in castings. High levels of carbon-silicate indicated that the castings would have low tensile strength.

We chose to apply the CUSUM control chart using many of the same initial cost and time parameter values. We made small changes to a number of the variables to obtain reasonable symmetric high and low levels for the designed experiment. We also included a nonzero fixed cost

per sample term. The high (low) levels for each variable were found by increasing (decreasing) the center point by about 30 percent. The design points are listed in the appendix. The center point levels are listed below.

| | | |
|---|---|---|
| $\lambda = 0.03$ | $E = 0.333$ | a = \$1.0 |
| $T_0 = 0.333$ | $C_0$ = \$115 | b = \$4.0 |
| $T_1 = 0.333$ | $C_1$ = \$950 | Y = \$975 |
| $T_2=1.5$ | W = \$975 | $\Delta = 0.75$ |

The runs were made using the algorithm previously described on a 486DX-33 personal computer with each optimization taking about three minutes. The $2^{12-6}$ fractional design including the center point resulted in 65 runs per scenario. The analysis consisted of determining the significant variables for each of the decision variables.

**Table 3.**
**Example 1 results (shift of 0.25-1.25)**

The results show that four of the twelve inputs significantly drive the cost response. The significant variables include $\lambda$, $C_0$, $C_1$, and $\Delta$. As the number of assignable causes per hour increases, cost also increases. The two quality cost variables, $C_0$ and $C_1$ are also positively correlated with cost. The process shift has a negative correlation, meaning that it costs more to detect smaller shifts. This four variable model accounts for over 90% of the total variability in the cost equation.

The CUSUM reference value (k) is almost entirely dependent on the level of the process shift. This result is consistent with practical guidelines that suggest setting the reference value equal to ½ the process shift to be detected [21 and 22]. According to Chiu [7] and Lucas [2], this approach gives the smallest out-of-control ARL (ARL2) for a given in-control ARL (ARL1). Because the optimal

cost control chart does not exclusively consider statistical performance, the resulting control chart reference values did not always equal $\Delta/2$, but were some function of $\Delta$.

The sensitivity models for the design parameters of decision interval, sampling interval, and sample size captured only about two-thirds of the response variability. Because the LV equation contains many occurrences of these responses both directly and indirectly (via ARL values), we suspected that many terms influenced these optimal cost design parameters. However, it was important to only select the largely significant variables because we were interested in a parsimonious model describing the underlying relationship between the responses and the influential inputs. The results for the CUSUM decision interval (H) indicated that $\Delta$ was the primary influence and Y, the cost of a false alarm, also had an affect. For the sampling interval response, we expected the LV model cost of sampling ratio term to have an impact. Indeed, the major contributors included three terms from that ratio $\lambda$, E, and b and a fourth term $C_1$, the cost while producing out-of-control. The sample size was a function of E and $\Delta$. In our ANOVA model development, we sometimes encountered unequal error variances, requiring some transformation of the response variable. In each case a logarithmic transformation worked well. We have indicated the models requiring transformations in the tables.

Although the study included five response variables, the most important variable was expected cost per time unit. The results of this example indicate that only four inputs significantly drove the cost response. If this result can be generalized for the LV model using the CUSUM chart, the practitioner's emphasis can be directed toward accurate estimation of this reduced set of variables. The next example will be used to test the generalization.

Example 2

We used a modification of the example shown in [21, p. 420] to test the sensitivity analysis results using somewhat different input values. In this example, control of soft drink bottle thickness is

monitored because the manufacturer is interested in detecting whether the wall of the glass is too thin. If this condition occurs the internal pressure generated during filling will cause the bottle to burst. The Montgomery example used an X-bar chart and applied Duncan's model that assumes the process continues during search and repair of the assignable cause. We used a CUSUM chart and, in the LV model, set the flags ($\delta_1$ and $\delta_2$) to one to simulate the process continuing during search and repair. We also assumed a nominal value for the defect cost during in-control condition. The process cost and time values used as the center points were developed from the example and the high and low values were obtained using the same range (+/- 30%) as Example 1. The center points are listed below.

| | | |
|---|---|---|
| $\lambda = 0.05$ | $E = 0.0833$ | $a = \$1.00$ |
| $T_0 = 1.0$ | $C_0 = \$5$ | $b = \$0.10$ |
| $T_1 = 1.0$ | $C_1 = \$100$ | $Y = \$50$ |
| $T_2 = 1.0$ | $W = \$25$ | $\Delta = 0.75$ |

This example provided an opportunity to test the diversity of the sensitivity results from the first example. It is important to compare the magnitudes of certain cost ratios between examples when searching for significantly different inputs. Some practical cost ratios include the ratio of cost to locate and repair the assignable cause to the quality cost per hour while producing out-of-control ($W/C_1$). The ratio of quality cost per hour while producing out-of-control to the quality cost per hour while producing in-control ($C_1/C_0$) is also a practical consideration and may intuitively have an impact on input variable significance. Table 4 shows that several of these ratios were compared for the two examples and the results indicated that the examples were different.

**Table 4.**
**Cost Ratios**

The results of this example are shown in Table 5 listed with the LV example results. Many of the significant variables from the LV example are also significant in the Montgomery example. For the cost response, the only change from the Example 1 results was that $C_0$ was not significant. For the reference value response, $\Delta$ was again the only significant variable, accounting for 99% of the variability. In the decision interval (H) model, two additional variables (a and b) were significant, indicating that sampling costs also affected H. Additional variables were also significant in the sampling interval model. The sample size model consisted of three terms E, $C_1$ and $\Delta$ that accounted for 91% of the variability.

**Table 5.**
**Example 1 and 2 results (shift of 0.25-1.25)**

The following suggestions are made based on the results from both examples. For the cost response it makes sense to seriously consider all four inputs ($\lambda$, $C_0$, $C_1$, and $\Delta$) significant in the first example. The process shift $\Delta$ drives the reference value. The major inputs in determining the decision interval H are Y and $\Delta$. The inputs E, $C_1$ and b were significant in both sampling interval (h) models. The combined results of the sampling interval and sample size models indicate that as E and $C_1$ decrease, the decision variables h and n increase.

## Small vs. Large Process Shifts

We pointed out earlier that the two examples contained significantly different input values. The only independent variable with identical values between examples was $\Delta$. Because we chose the CUSUM for the sensitivity, small to medium shifts (0.25, 0.75 and 1.25), were used in the designs because the CUSUM is better than the Shewhart in detecting small shifts. We were interested if the results of the sensitivity would change if $\Delta$ represented medium to large shifts (1.25, 1.75, and

2.25). We selected the Lorenzen and Vance example (Example 1) to perform another $2^{12-6}$ design with a single center point run. The results are shown in Table 6 together with the small shift case.

**Table 6.**
**Example 1 using small and large process shifts**

For the cost response, the size of the shift makes no difference in terms of important variables. Both models accounted for over 90% of the variability. The larger process shift still drove the reference value. The results for the other decision variables were similar to the small shift case, with minor changes in two of the models. As was the case for the examples in the small shift scenario, $\Delta$ was significant in each model that was developed for the large shift scenario. These results are not surprising, but they emphasize the importance of correctly specifying the size of the process shift.

Verification

Although the results for the two control chart variables (k and H), the sample size, and the sampling interval are interesting and can be beneficial in the decision making process, often the primary focus in economic design is the overall process cost. We decided to develop a third example, different from the other two, to test the results of the sensitivity analysis. A verification of the significant variables was performed by comparing the range of optimal costs for non-significant variable fluctuation (+/- 30%) versus the optimal cost range for significant variable cost fluctuation. We first ran a baseline case using the following input variable values.

| | | |
|---|---|---|
| $\lambda = 0.067$ | $E = 0.10$ | $a = \$0.30$ |
| $T_0 = 0.6$ | $C_0 = \$10$ | $b = \$0.10$ |
| $T_1 = 0.3$ | $C_1 = \$50$ | $Y = \$20$ |
| $T_2 = 0.2$ | $W = \$10$ | $\Delta = 0.75$ |

The expected relation between each input variable and the response was determined by studying the LV cost function. For instance, it is clear from (1) that as the fixed cost per sample (a) increases, total cost will increase. The relation between cost and other terms such as the expected time to discover the assignable cause ($T_1$), depend on the value assigned to the process continue or cease flags ($\delta_1$ and $\delta_2$).

**Table 7.**
**Results of verification test**

Based on the results of the two example sensitivity analysis, three variables ($\lambda$, $C_1$, and $\Delta$) were significant in both examples and $C_0$ was significant in only the first example. For this demonstration, all four variables were labeled significant. The remaining eight inputs comprised the non-significant group. The significant variables were then modified by 30% in the direction of increasing cost, while the non-significant variables were held constant. The optimal cost was recorded. Then the significant variables were modified in the direction of decreasing cost again holding the others constant and the optimal cost was recorded. The difference between the costs was calculated to determine the impact the significant variables had on cost variability. A similar experiment was conducted altering the nonsignificant variables, holding the significant variables constant. The difference in costs for the nonsignificant variables was compared to the significant variable cost variability. The results in Table 7 show that the four significant variables represent an 85% change in optimal cost while the eight nonsignificant variables only alter the baseline cost by 21%.

Misspecification

In effort to determine the impact that inaccurate estimation of the input variables have on optimal cost, we developed a scenario using the verification test baseline and altered the variables in groups

(significant, then nonsignificant) by +/- 10%. Table 8 shows that modifying the non-significant variables over this range of 20% only changes optimal cost by 7%, but misspecifying the significant variables by the same amount affects cost by 24%. Obviously the emphasis on accurate estimation should be placed on the four significant inputs.

**Table 8.**
**Results of misspecification test**

## Conclusion

The sensitivity analysis was designed to provide insight into the significant inputs to the LV model when the CUSUM control chart is employed. By restricting our analysis to highly significant factors only, the four main effects: rate of shift, magnitude of shift, and the costs while producing *in and out-of-control, have the largest effect on the dependent* variable, expected cost per time unit. We have also identified key input variables with respect to the decision variables of the LV model, control limit, reference value, sample size and time between samples. We have verified our conclusions concerning highly significant variables with respect to expected cost per time unit by changing and not changing the highly significant variables and noting the effects on expected cost.

A major obstacle to industrial implementation of the LV model is the large numbers of terms and difficulties in their estimation. Our results indicate that one could use as few as four input variables and observe relatively small changes in the cost response relative to the full model. This study provides a basis for the investigation of the use of cost ratios rather than actual cost as a further aid to implementation.

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| *Symbol* | *Description* |
|---|---|
| $\lambda$ | Number of assignable causes per hour |
| $T_0$ | Expected search time when false alarm |
| $T_1$ | Expected time to discover the assignable cause |
| $T_2$ | Expected time to repair the process |
| E | Time to sample and chart one item |
| $C_0$ | Quality cost/hr while producing in control |
| $C_1$ | Quality cost/hr while producing out-of-control |
| W | Cost for search/repair |
| a | Fixed cost per sample |
| b | Variable cost per unit |
| Y | Cost per false alarm |
| $\Delta$ | Mean shift - number of standard deviations slip when out-of-control |
| n | Sample size |
| h | Hours between samples or sampling interval |
| k | CUSUM reference value |
| H | CUSUM decision interval |
| $\delta_1$ | Flag for whether production continues during searches (1-yes, 0-no) |
| $\delta_2$ | Flag for whether production continues during repairs (1-yes, 0-no) |
| ARL1 | In-control average run length |
| ARL2 | Out-of-control average run length |
| L | X bar chart - number of standard deviations from control limits to center line |
| $\alpha$ | Probability of a type I error or probability of a false alarm |
| $\beta$ | Probability of a type II error or 1.0 minus the power of the test |

**Table 1.**
**Explanation of Model Terms**

| | | *LV Model Component* | | |
|---|---|---|---|---|
| *Symbol* | *Description* | *Cost of producing bad product* | *Cost of search and repair (true cause and false alarms)* | *Cost of sampling* |
| $\lambda$ | Number of assignable causes per hour | x | x | x |
| $T_0$ | Expected search time when false alarm | x | x | x |
| $T_1$ | Expected time to discover the assignable cause | x | x | x |
| $T_2$ | Expected time to repair the process | x | x | x |
| E | Time to sample and chart one item | x | x | x |
| $C_0$ | Quality cost/hr while producing in control | x | | |
| $C_1$ | Quality cost/hr while producing out-of-control | x | | |
| W | Cost for search/repair | | x | |
| a | Fixed cost per sample | | | x |
| b | Variable cost per unit | | | x |
| Y | Cost per false alarm | | x | |
| $\Delta$ | Mean shift - number of standard deviations slip when out-of-control | x | x | x |

**Table 2.**
**Input Variables**

| *Response* | Transform? | $R^2$ | $\lambda$ | E | $C_0$ | $C_1$ | a | b | Y | $\Delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| *Cost* | | 0.91 | + | | + | + | | | | - |
| *Reference Value* | | 0.88 | | | | | | | | + |
| *Decision Interval* | | 0.67 | | | | | | | + | - |
| *Sampling Interval* | log e | 0.64 | - | - | | - | | + | | |
| *Sample Size* | log e | 0.67 | | - | | | | | | - |

**Table 3.**
**Example 1 results (shift of 0.25-1.25)**

| | $W / C_1$ | $C_1 / C_0$ | $C_1 / b$ | $W / Y$ |
|---|---|---|---|---|
| Example 1 | 1.03 | 8.3 | 237.5 | 1.0 |
| Example 2 | 0.25 | 20.0 | 1000.0 | 0.5 |

**Table 4.**
**Cost Ratios**

| *Response* | Transform? | $R^2$ | $\lambda$ | E | $C_0$ | $C_1$ | a | b | Y | $\Delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| ***Cost*** | | | | | | | | | | |
| LV example | | 0.91 | + | | + | + | | | | - |
| Ex. 10-1 | | 0.91 | + | | | + | | | | - |
| ***Reference Value*** | | | | | | | | | | |
| LV example | | 0.88 | | | | | | | | + |
| Ex. 10-1 | | 0.99 | | | | | | | | + |
| ***Decision Interval*** | | | | | | | | | | |
| LV example | | 0.67 | | | | | | | + | - |
| Ex. 10-1 | log e | 0.64 | | | | | - | - | + | - |
| ***Sampling Interval*** | | | | | | | | | | |
| LV example | log e | 0.64 | - | - | | - | | + | | |
| Ex. 10-1 | | 0.58 | | - | | - | + | + | - | + |
| ***Sample Size*** | | | | | | | | | | |
| LV example | log e | 0.67 | | - | | | | | | - |
| Ex. 10-1 | | 0.91 | | - | | - | | | | - |

**Table 5.**
**Example 1 and 2 results (shift of 0.25-1.25)**

| *Responses* | Transform? | $R^2$ | $\lambda$ | E | $C_0$ | $C_1$ | b | Y | $\Delta$ |
|---|---|---|---|---|---|---|---|---|---|
| ***Cost*** | | | | | | | | | |
| Small Shift | | 0.91 | + | | + | + | | | - |
| Large Shift | | 0.90 | + | | + | + | | | - |
| ***Reference Value*** | | | | | | | | | |
| Small Shift | | 0.88 | | | | | | | + |
| Large Shift | | 0.80 | | | | | | | + |
| ***Decision Interval*** | | | | | | | | | |
| Small Shift | | 0.67 | | | | | | + | - |
| Large Shift | | 0.60 | | | | | - | | - |
| ***Sampling Interval*** | | | | | | | | | |
| Small Shift | log e | 0.64 | - | - | | - | + | | |
| Large Shift | log e | 0.76 | - | | | - | + | | + |
| ***Sample Size*** | | | | | | | | | |
| Small Shift | log e | 0.67 | | - | | | | | - |
| Large Shift | | - | insufficient variability - all but 8 runs had n=1 as optimum sample size | | | | | | |

**Table 6.**
**Example 1 using small and large process shifts**

| | Input variables | | | | | | | | | | | | | Results | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\lambda$ | $T_0$ | $T_1$ | $T_2$ | E | $C_0$ ($) | $C_1$ ($) | W ($) | a ($) | b ($) | Y ($) | $\Delta$ | | Cost ($) | % change over base |
| expected relation with cost | + | - | - | - | + | + | + | + | + | + | + | - | | | |
| | | | | | | | | | | | | | | | |
| baseline | 0.067 | 0.6 | 0.3 | 0.2 | 0.10 | 10 | 50 | 10 | 0.3 | 0.10 | 20 | 0.75 | | 16.07 | |
| | | | | | | | | | | | | | | | |
| modify non-significant variables | | | | | | | | | | | | | | | |
| maximize cost | 0.067 | 0.42 | 0.21 | 0.14 | 0.13 | 10 | 50 | 13 | 0.39 | 0.13 | 26 | 0.75 | | 17.56 | |
| minimize cost | 0.067 | 0.78 | 0.39 | 0.26 | 0.07 | 10 | 50 | 7 | 0.21 | 0.07 | 14 | 0.75 | | 14.10 | ***21%*** |
| | | | | | | | | | | | | | | | |
| modify significant variables | | | | | | | | | | | | | | | |
| maximize cost | 0.087 | 0.6 | 0.3 | 0.2 | 0.10 | 13 | 65 | 10 | 0.3 | 0.10 | 20 | 0.25 | | 23.73 | |
| minimize cost | 0.047 | 0.6 | 0.3 | 0.2 | 0.10 | 7 | 35 | 10 | 0.3 | 0.10 | 20 | 1.25 | | 10.07 | ***85%*** |

**Table 7.**
**Results of verification test**

| | Input variables | | | | | | | | | | | | | Results | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\lambda$ | $T_0$ | $T_1$ | $T_2$ | E | $C_0$ ($) | $C_1$ ($) | W ($) | a ($) | b ($) | Y ($) | $\Delta$ | | Cost ($) | % change over base |
| expected relation with cost | + | - | - | - | + | + | + | + | + | + | + | - | | | |
| | | | | | | | | | | | | | | | |
| baseline | 0.07 | 0.6 | 0.3 | 0.2 | 0.1 | 10 | 50 | 10 | 0.3 | 0.1 | 20 | 0.75 | | 16.07 | |
| | | | | | | | | | | | | | | | |
| modify non-significant variables | | | | | | | | | | | | | | | |
| maximize cost | 0.07 | 0.54 | 0.27 | 0.18 | 0.11 | 10 | 50 | 11 | 0.33 | 0.11 | 22 | 0.75 | | 16.62 | |
| minimize cost | 0.07 | 0.66 | 0.33 | 0.22 | 0.09 | 10 | 50 | 9 | 0.27 | 0.09 | 18 | 0.75 | | 15.52 | *7%* |
| | | | | | | | | | | | | | | | |
| modify significant variables | | | | | | | | | | | | | | | |
| maximize cost | 0.07 | 0.6 | 0.3 | 0.2 | 0.1 | 11 | 55 | 10 | 0.3 | 0.1 | 20 | 0.68 | | 18.10 | |
| minimize cost | 0.06 | 0.6 | 0.3 | 0.2 | 0.1 | 9 | 45 | 10 | 0.3 | 0.1 | 20 | 0.83 | | 14.20 | *24%* |

**Table 8.**
**Results of misspecification test**

# Appendix

## Example 1 Design Points and Results

| Input variables | | | | | | | | | | | | Output variables | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| λ | T0 | T1 | T2 | E | C0 | C1 | W | A | B | Y | Δ | Cost | k | H | h | n | ARL1 | ARL2 |
| 0.04 | 0.23 | 0.23 | 1 | 0.23 | 80 | 1250 | 1275 | 0.7 | 2.8 | 675 | 0.25 | 382.61 | 0.25 | 5.5 | 0.15 | 3 | 93.78 | 22.81 |
| 0.02 | 0.23 | 0.43 | 2 | 0.43 | 150 | 1250 | 1275 | 0.7 | 2.8 | 1275 | 0.25 | 376.99 | 0.25 | 6.5 | 0.19 | 3 | 162.55 | 27.87 |
| 0.02 | 0.43 | 0.43 | 1 | 0.43 | 80 | 650 | 1275 | 1.3 | 2.8 | 1275 | 1.25 | 152.34 | 0.625 | 5 | 0.25 | 1 | 1369.79 | 8.64 |
| 0.02 | 0.23 | 0.43 | 2 | 0.23 | 80 | 1250 | 1275 | 1.3 | 5.2 | 1275 | 0.25 | 343.07 | 0.25 | 5.5 | 0.28 | 3 | 93.78 | 22.81 |
| 0.02 | 0.43 | 0.43 | 1 | 0.43 | 150 | 650 | 1275 | 1.3 | 5.2 | 675 | 0.25 | 305.84 | 0.125 | 5.5 | 0.46 | 2 | 41.81 | 20.07 |
| 0.02 | 0.43 | 0.43 | 2 | 0.43 | 150 | 650 | 675 | 0.7 | 2.8 | 675 | 1.25 | 198.31 | 0.625 | 4.5 | 0.26 | 1 | 735.97 | 7.84 |
| 0.02 | 0.23 | 0.23 | 2 | 0.23 | 150 | 650 | 1275 | 0.7 | 5.2 | 675 | 0.25 | 295.72 | 0.25 | 4 | 0.56 | 3 | 38.45 | 15.57 |
| 0.04 | 0.23 | 0.43 | 1 | 0.23 | 150 | 650 | 1275 | 1.3 | 2.8 | 1275 | 0.25 | 360.74 | 0.25 | 6.5 | 0.29 | 4 | 162.55 | 22.87 |
| 0.04 | 0.43 | 0.43 | 1 | 0.43 | 150 | 1250 | 675 | 0.7 | 2.8 | 1275 | 0.25 | 467.58 | 0.25 | 6.5 | 0.15 | 3 | 162.55 | 27.87 |
| 0.04 | 0.23 | 0.43 | 2 | 0.43 | 150 | 650 | 675 | 1.3 | 5.2 | 675 | 0.25 | 340.36 | 0.125 | 6.5 | 0.21 | 1 | 61.53 | 34.03 |
| 0.02 | 0.23 | 0.23 | 1 | 0.43 | 80 | 650 | 675 | 0.7 | 5.2 | 675 | 1.25 | 143.25 | 0.625 | 4 | 0.34 | 1 | 393.94 | 7.04 |
| 0.02 | 0.43 | 0.43 | 2 | 0.43 | 80 | 650 | 675 | 0.7 | 5.2 | 1275 | 0.25 | 251.14 | 0.25 | 6 | 0.29 | 2 | 123.88 | 33.13 |
| 0.02 | 0.23 | 0.23 | 2 | 0.23 | 80 | 650 | 1275 | 0.7 | 2.8 | 1275 | 1.25 | 143.05 | 0.875 | 3.5 | 0.42 | 2 | 1094.70 | 4.67 |
| 0.02 | 0.43 | 0.43 | 1 | 0.23 | 80 | 650 | 1275 | 0.7 | 2.8 | 675 | 0.25 | 226.19 | 0.25 | 5.5 | 0.30 | 3 | 93.78 | 22.81 |
| 0.04 | 0.23 | 0.23 | 1 | 0.43 | 80 | 1250 | 1275 | 1.3 | 2.8 | 1275 | 1.25 | 233.72 | 0.625 | 5 | 0.15 | 1 | 1369.79 | 8.64 |
| 0.02 | 0.23 | 0.23 | 2 | 0.43 | 150 | 650 | 1275 | 1.3 | 5.2 | 1275 | 1.25 | 219.24 | 0.625 | 4.5 | 0.37 | 1 | 735.97 | 7.84 |
| 0.04 | 0.43 | 0.23 | 2 | 0.43 | 150 | 650 | 1275 | 0.7 | 2.8 | 1275 | 0.25 | 351.92 | 0.25 | 6.5 | 0.23 | 3 | 162.55 | 27.87 |
| 0.04 | 0.23 | 0.23 | 1 | 0.23 | 150 | 1250 | 1275 | 0.7 | 5.2 | 1275 | 1.25 | 300.97 | 0.625 | 5 | 0.15 | 1 | 1369.79 | 8.64 |
| 0.02 | 0.43 | 0.23 | 2 | 0.43 | 150 | 1250 | 675 | 1.3 | 5.2 | 675 | 0.25 | 368.40 | 0.125 | 6.5 | 0.17 | 1 | 61.53 | 34.03 |
| 0.04 | 0.23 | 0.23 | 2 | 0.23 | 80 | 1250 | 675 | 1.3 | 5.2 | 675 | 1.25 | 206.73 | 0.625 | 4 | 0.18 | 1 | 393.94 | 7.04 |
| 0.04 | 0.43 | 0.23 | 2 | 0.43 | 80 | 650 | 1275 | 0.7 | 5.2 | 675 | 1.25 | 190.53 | 0.625 | 4 | 0.25 | 1 | 393.94 | 7.04 |
| 0.04 | 0.43 | 0.43 | 2 | 0.43 | 150 | 1250 | 1275 | 1.3 | 5.2 | 1275 | 1.25 | 307.47 | 0.625 | 4.5 | 0.18 | 1 | 735.97 | 7.84 |
| 0.02 | 0.23 | 0.23 | 1 | 0.43 | 150 | 650 | 675 | 0.7 | 2.8 | 1275 | 0.25 | 290.05 | 0.25 | 6.5 | 0.29 | 3 | 162.55 | 27.87 |
| 0.02 | 0.43 | 0.43 | 2 | 0.23 | 80 | 650 | 675 | 1.3 | 5.2 | 675 | 1.25 | 141.43 | 0.875 | 2.5 | 0.72 | 2 | 189.01 | 3.55 |
| 0.04 | 0.23 | 0.23 | 2 | 0.23 | 150 | 1250 | 675 | 1.3 | 2.8 | 1275 | 0.25 | 435.67 | 0.25 | 6.5 | 0.15 | 3 | 162.55 | 27.87 |
| 0.02 | 0.43 | 0.23 | 1 | 0.43 | 80 | 1250 | 1275 | 0.7 | 5.2 | 1275 | 0.25 | 350.34 | 0.25 | 6.5 | 0.17 | 2 | 162.55 | 36.91 |
| 0.04 | 0.43 | 0.43 | 2 | 0.23 | 80 | 1250 | 1275 | 0.7 | 2.8 | 1275 | 1.25 | 221.91 | 0.625 | 5 | 0.15 | 1 | 1369.79 | 8.64 |
| 0.02 | 0.43 | 0.23 | 1 | 0.43 | 150 | 1250 | 1275 | 0.7 | 2.8 | 675 | 1.25 | 236.22 | 0.625 | 4.5 | 0.17 | 1 | 735.97 | 7.84 |
| 0.04 | 0.43 | 0.43 | 2 | 0.43 | 80 | 1250 | 1275 | 1.3 | 2.8 | 675 | 0.25 | 393.49 | 0.25 | 5 | 0.15 | 2 | 70.42 | 25.93 |
| 0.02 | 0.43 | 0.43 | 1 | 0.23 | 150 | 650 | 1275 | 0.7 | 5.2 | 1275 | 1.25 | 221.28 | 0.875 | 3 | 0.67 | 2 | 456.84 | 4.12 |
| 0.02 | 0.23 | 0.23 | 1 | 0.23 | 80 | 650 | 675 | 1.3 | 5.2 | 1275 | 0.25 | 251.30 | 0.25 | 5.5 | 0.42 | 3 | 93.78 | 22.81 |
| 0.02 | 0.43 | 0.43 | 2 | 0.23 | 150 | 650 | 675 | 1.3 | 2.8 | 1275 | 0.25 | 283.93 | 0.25 | 6.5 | 0.36 | 4 | 162.55 | 22.87 |
| 0.02 | 0.23 | 0.23 | 1 | 0.23 | 150 | 650 | 675 | 1.3 | 2.8 | 675 | 1.25 | 199.44 | 0.875 | 3 | 0.51 | 2 | 456.84 | 4.12 |
| 0.03 | 0.33 | 0.33 | 1.5 | 0.33 | 115 | 950 | 975 | 1 | 4 | 975 | 0.75 | 247.69 | 0.375 | 6.5 | 0.15 | 1 | 529.72 | 16.95 |
| 0.04 | 0.23 | 0.23 | 2 | 0.43 | 150 | 1250 | 675 | 0.7 | 2.8 | 675 | 1.25 | 257.42 | 0.625 | 4.5 | 0.15 | 1 | 735.97 | 7.84 |
| 0.02 | 0.23 | 0.43 | 2 | 0.43 | 80 | 1250 | 1275 | 0.7 | 5.2 | 675 | 1.25 | 182.94 | 0.625 | 4 | 0.24 | 1 | 393.94 | 7.04 |
| 0.04 | 0.23 | 0.43 | 1 | 0.43 | 80 | 650 | 1275 | 0.7 | 5.2 | 1275 | 0.25 | 337.30 | 0.25 | 6 | 0.23 | 2 | 123.88 | 33.13 |
| 0.02 | 0.43 | 0.23 | 2 | 0.43 | 80 | 1250 | 675 | 1.3 | 2.8 | 1275 | 1.25 | 162.07 | 0.625 | 5 | 0.17 | 1 | 1369.79 | 8.64 |
| 0.02 | 0.43 | 0.23 | 2 | 0.23 | 80 | 1250 | 675 | 0.7 | 2.8 | 675 | 0.25 | 273.88 | 0.25 | 6 | 0.17 | 3 | 123.88 | 25.33 |
| 0.04 | 0.43 | 0.23 | 2 | 0.23 | 150 | 650 | 1275 | 1.3 | 2.8 | 675 | 1.25 | 239.29 | 0.875 | 3 | 0.37 | 2 | 456.84 | 4.12 |
| 0.04 | 0.43 | 0.23 | 1 | 0.43 | 150 | 650 | 675 | 1.3 | 5.2 | 1275 | 1.25 | 243.26 | 0.625 | 4.5 | 0.26 | 1 | 735.97 | 7.84 |
| 0.02 | 0.43 | 0.23 | 2 | 0.23 | 150 | 1250 | 675 | 0.7 | 5.2 | 1275 | 1.25 | 232.11 | 0.625 | 5 | 0.20 | 1 | 1369.79 | 8.64 |
| 0.04 | 0.43 | 0.23 | 1 | 0.23 | 150 | 650 | 675 | 0.7 | 5.2 | 675 | 0.25 | 342.91 | 0.125 | 5.5 | 0.34 | 2 | 41.81 | 20.07 |
| 0.02 | 0.43 | 0.23 | 1 | 0.23 | 150 | 1250 | 1275 | 1.3 | 2.8 | 1275 | 0.25 | 373.58 | 0.25 | 6.5 | 0.23 | 4 | 162.55 | 22.87 |
| 0.04 | 0.43 | 0.43 | 2 | 0.23 | 150 | 1250 | 1275 | 0.7 | 5.2 | 675 | 0.25 | 461.57 | 0.125 | 6 | 0.19 | 2 | 50.96 | 22.17 |
| 0.04 | 0.23 | 0.23 | 1 | 0.43 | 150 | 1250 | 1275 | 1.3 | 5.2 | 675 | 0.25 | 481.49 | 0.125 | 6 | 0.15 | 1 | 50.96 | 30.66 |
| 0.04 | 0.23 | 0.43 | 2 | 0.43 | 80 | 650 | 675 | 1.3 | 2.8 | 1275 | 1.25 | 168.39 | 0.625 | 5 | 0.18 | 1 | 1369.79 | 8.64 |
| 0.02 | 0.23 | 0.43 | 1 | 0.43 | 150 | 1250 | 675 | 1.3 | 5.2 | 1275 | 1.25 | 246.88 | 0.625 | 4.5 | 0.24 | 1 | 735.97 | 7.84 |
| 0.02 | 0.23 | 0.43 | 1 | 0.23 | 80 | 1250 | 675 | 0.7 | 2.8 | 1275 | 1.25 | 161.46 | 0.625 | 5.5 | 0.15 | 1 | 2542.24 | 9.45 |
| 0.02 | 0.43 | 0.23 | 1 | 0.23 | 80 | 1250 | 1275 | 1.3 | 5.2 | 675 | 1.25 | 180.61 | 0.625 | 4 | 0.24 | 1 | 393.94 | 7.04 |
| 0.04 | 0.43 | 0.43 | 1 | 0.43 | 80 | 1250 | 675 | 0.7 | 5.2 | 675 | 1.25 | 226.21 | 0.625 | 4 | 0.17 | 1 | 393.94 | 7.04 |
| 0.04 | 0.43 | 0.43 | 1 | 0.23 | 150 | 1250 | 675 | 1.3 | 2.8 | 675 | 1.25 | 270.55 | 0.625 | 4.5 | 0.15 | 1 | 735.97 | 7.84 |
| 0.04 | 0.23 | 0.43 | 2 | 0.23 | 80 | 650 | 675 | 0.7 | 2.8 | 675 | 0.25 | 263.16 | 0.25 | 5.5 | 0.22 | 3 | 93.78 | 22.81 |
| 0.04 | 0.43 | 0.23 | 2 | 0.23 | 80 | 650 | 1275 | 1.3 | 5.2 | 1275 | 0.25 | 322.71 | 0.25 | 5 | 0.37 | 3 | 70.42 | 20.35 |
| 0.02 | 0.23 | 0.43 | 2 | 0.23 | 150 | 1250 | 1275 | 1.3 | 2.8 | 675 | 1.25 | 234.64 | 0.875 | 3 | 0.34 | 2 | 456.84 | 4.12 |
| 0.02 | 0.23 | 0.43 | 1 | 0.23 | 150 | 1250 | 675 | 0.7 | 5.2 | 675 | 0.25 | 369.95 | 0.125 | 6 | 0.25 | 2 | 50.96 | 22.17 |
| 0.04 | 0.23 | 0.23 | 2 | 0.43 | 80 | 1250 | 675 | 0.7 | 5.2 | 1275 | 0.25 | 429.27 | 0.25 | 6 | 0.15 | 2 | 123.88 | 33.13 |
| 0.02 | 0.23 | 0.23 | 2 | 0.43 | 80 | 650 | 1275 | 1.3 | 2.8 | 675 | 0.25 | 227.40 | 0.25 | 5.5 | 0.30 | 3 | 93.78 | 22.81 |
| 0.04 | 0.43 | 0.43 | 1 | 0.23 | 80 | 1250 | 675 | 1.3 | 5.2 | 1275 | 0.25 | 440.90 | 0.25 | 5.5 | 0.21 | 3 | 93.78 | 22.81 |
| 0.04 | 0.43 | 0.23 | 1 | 0.43 | 80 | 650 | 675 | 1.3 | 2.8 | 675 | 0.25 | 277.63 | 0.25 | 5.5 | 0.18 | 2 | 93.78 | 29.47 |
| 0.04 | 0.43 | 0.23 | 1 | 0.23 | 80 | 650 | 675 | 0.7 | 2.8 | 1275 | 1.25 | 163.83 | 0.625 | 5.5 | 0.15 | 1 | 2542.24 | 9.45 |
| 0.04 | 0.23 | 0.43 | 1 | 0.23 | 80 | 650 | 1275 | 1.3 | 5.2 | 675 | 1.25 | 199.23 | 0.875 | 2.5 | 0.53 | 2 | 189.01 | 3.55 |
| 0.04 | 0.23 | 0.43 | 1 | 0.43 | 150 | 650 | 1275 | 0.7 | 2.8 | 675 | 1.25 | 251.73 | 0.625 | 4.5 | 0.20 | 1 | 735.97 | 7.84 |
| 0.02 | 0.23 | 0.43 | 1 | 0.43 | 80 | 1250 | 675 | 1.3 | 2.8 | 675 | 0.25 | 293.86 | 0.25 | 6 | 0.15 | 2 | 123.88 | 33.13 |
| 0.04 | 0.23 | 0.43 | 2 | 0.23 | 150 | 650 | 675 | 0.7 | 5.2 | 1275 | 1.25 | 232.86 | 0.625 | 4.5 | 0.25 | 1 | 735.97 | 7.84 |